



**General Certificate of Education**

**Mathematics 6360**

**MS03      Statistics 3**

**Mark Scheme**

*2009 examination - June series*

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## Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√ or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A <sub>2,1</sub>	2 or 1 (or 0) accuracy marks	NOS	not on scheme
-x EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

### No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

**Otherwise we require evidence of a correct method for any marks to be awarded.**

## MS03

Q	Solution	Marks	Total	Comments
1(a)	$\hat{p}_1 = \frac{102}{150} = 0.68$	B1		Both CAO
	$\hat{p}_2 = \frac{36}{80} = 0.45$			
	99% (0.99) $\Rightarrow z = 2.57$ to 2.58	B1		AWFW (2.5758)
	CI for $(p_1 - p_2)$ is			
	$(\hat{p}_1 - \hat{p}_2) \pm z \times \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$	M1 m1		Use of $(\hat{p}_1 - \hat{p}_2) \pm z \times \sqrt{\text{attempted variance}}$ Use of correct expression for variance
Thus $(0.68 - 0.45) \pm 2.5758 \times$				
$\sqrt{\frac{0.68 \times 0.32}{150} + \frac{0.45 \times 0.55}{80}}$	A1F		F on $\hat{p}_1$ , $\hat{p}_2$ and $z$	
Hence $0.23 \pm (0.173 \text{ to } 0.174)$				
<b>or</b> $(0.056 \text{ to } 0.057, 0.403 \text{ to } 0.404)$	A1	6	CAO & AFWF (accept 0.17) AWFW (accept 0.06 & 0.4)	
(b)	Whole of confidence interval is <b>above 0 or zero</b>	B1F		F on (a) Or equivalent
	so <b>Disagree</b> with claim / claim appears <b>doubtful</b>	B1F	2	F on (a) Or equivalent Dependent on previous B1F
	<b>Total</b>		<b>8</b>	

## MS03 (cont)

Q	Solution	Marks	Total	Comments
2(a)(i)	$P(B \& B) = (0.30 \times 0.80) + (0.55 \times 0.10) + (0.15 \times 0.30)$	M1		Use of <b>3 possibilities</b> each the product of <b>2 probabilities</b>
	$= 0.24 + 0.055 + 0.045 = 0.34$	A1	2	CAO; <b>AG</b>
(ii)	$P(HB \cap \text{Coastal}) = 0.55 \times 0.65$	M1		Can be implied by <b>correct</b> answer
	$= 143/400$ or 0.357 to 0.358	A1	2	CAO/AWFW (0.3575)
(iii)	$P(\text{Coastal}   HB) = \frac{P(\text{Coastal} \cap HB)}{P(HB)}$	M1		answer to (ii)
	$= \frac{0.3575}{(0.3 \times 0.15) + (0.3575) + (0.15 \times 0.5)}$	M1		$\sum(3 \times 2)$ probabilities
	$= \frac{0.3575}{0.4775} = 143/191$ or 0.747 to 0.75	A1F		F on (ii)
	$= \frac{0.3575}{0.4775} = 143/191$ or 0.747 to 0.75	A1	4	CAO/AWFW (0.74869)
(b)	$P(\text{City}   HB) = \frac{0.3 \times 0.15}{P(HB)} = \frac{0.045}{0.4775} = \frac{90}{955}$	M1		
	$P(\text{Country}   HB) = \frac{0.15 \times 0.5}{P(HB)} = \frac{0.075}{0.4775} = \frac{30}{191}$	M1		Or $\left(1 - (a)(iii) - \frac{0.045}{0.4775}\right)$
	Thus Probability = $\frac{0.045}{P(HB)} \times \frac{0.3575}{P(HB)} \times \frac{0.075}{P(HB)}$	M1		Multiplication of 3 different probabilities
	Multiplied by $3! = 6$	B1		CAO
	$= 0.09424 \times 0.74869 \times 0.15707 \times 6$ $= 0.063$ to $0.068$	A1	5	AWFW (0.06649)
	<b>Total</b>		<b>13</b>	

**MS03 (cont)**

<b>Q</b>	<b>Solution</b>	<b>Marks</b>	<b>Total</b>	<b>Comments</b>
<b>3</b>	<p>98% (0.98) CI <math>\Rightarrow z = 2.32</math> to <math>2.33</math></p> <p>CI width is <math>2 \times z \times \sqrt{\frac{p(1-p)}{n}}</math></p> <p><math>p = 0.35</math> or <math>0.50</math></p> <p>Thus <math>2 \times 2.3263 \times \sqrt{\frac{0.35 \times 0.65}{n}} = 0.1</math></p> <p>Thus <math>\sqrt{n} = \frac{2 \times 2.3263}{0.1} \times \sqrt{0.35 \times 0.65}</math></p> <p>Thus <math>n = 492.5</math> (<math>p = 0.35</math>) or <math>n = 541.2</math> (<math>p = 0.50</math>)</p> <p>Thus to nearest 10 <math>n = 500</math> or <math>490</math></p> <p><b>Notes:</b> No '<math>\times 2</math>' gives <math>n = 123.1</math> No '<math>\times 2</math>' and <math>p = 0.50</math> gives <math>n = 135.3</math></p>	<p>B1</p> <p>M1</p> <p>B1</p> <p>A1F</p> <p>m1</p> <p>A1</p>	<b>6</b>	<p>AWFW (2.3263)</p> <p>Used; allow <math>z \times \sqrt{\frac{p(1-p)}{n}}</math></p> <p>Or equivalent F on <math>z</math>; allow no multiplier of 2 and/or <math>p = 0.50</math></p> <p>Solving for <math>\sqrt{n}</math> or <math>n</math></p> <p>Either</p>
	<b>Total</b>		<b>6</b>	

## MS03 (cont)

Q	Solution	Marks	Total	Comments
4	$H_0: \mu_X - \mu_Y = 15$	B1		Or equivalent Accept $H_0: \mu_X - \mu_Y = 0$
	$H_1: \mu_X - \mu_Y > 15$	B1		Or equivalent
	SL $\alpha = 1\% (0.01)$			
	CV $z = 2.32$ to 2.33	B1		AWFW (2.3263) If $H_1$ involves ' $\neq$ ' then accept 2.57 to 2.58 (2.5758)
	CV $t = 2.35$ to 2.36	(B1)		AWFW If $H_1$ involves ' $\neq$ ' then accept 2.60 to 2.62
	$z = \frac{(\bar{x} - \bar{y}) - 15}{\sqrt{\frac{s_X^2}{n_X} + \frac{s_Y^2}{n_Y}}}$ or $z/t = \frac{(\bar{x} - \bar{y}) - 15}{\sqrt{s_p^2 \left( \frac{1}{n_X} + \frac{1}{n_Y} \right)}}$	M1		Used Allow 'no -15'
	$s_p^2 = \frac{(64 \times 3.4^2) + (74 \times 2.8^2)}{65 + 75 - 2}$			
	$= \frac{1320}{138} = 9.56522$			$s_p = 3.09277$
	$z = \frac{(40.7 - 24.4) - 15}{\sqrt{\frac{3.4^2}{65} + \frac{2.8^2}{75}}} = \frac{1.3}{\sqrt{0.28238}}$	A1		Numerator; allow 'no -15'
		A1		Denominator
	$= 2.44$ to 2.45	A1		AWFW (2.4464) 'no -15' gives $z = 30.674$
	<b>OR</b>			
	$z/t = \frac{(40.7 - 24.4) - 15}{\sqrt{\frac{1320}{138} \left( \frac{1}{65} + \frac{1}{75} \right)}} = \frac{1.3}{\sqrt{0.27469}}$	(A1)		Numerator; allow 'no -15'
	(A1)		Denominator	
$= 2.48$	(A1)		AWRT (2.4804) 'no -15' gives $z = 31.100$	
Thus evidence, at 1% level, to support Holly's belief	A1F	8	F on $z$ and CV	
	<b>Total</b>		<b>8</b>	

## MS03 (cont)

Q	Solution	Marks	Total	Comments
5	$X \sim B(n, p)$			
(a)	$\text{Var}(X) = E(X^2) - [E(X)]^2$ $= E[X(X-1)] + E(X) - [E(X)]^2$ $= n(n-1)p^2 + np - n^2p^2$ $= np - np^2 = np(1-p)$	M1 M1 A1		Used; may be implied Rearranging & substitution Or equivalent
	<b>OR</b>			
	$E[X(X-1)] = E(X^2) - E(X)$ $= n(n-1)p^2 = n^2p^2 - np^2$	(M1)		Expansion & substitution
	$\text{Var}(X) = E(X^2) - [E(X)]^2$ $= \{n^2p^2 - np^2 + E(X)\} - n^2p^2$ $= np - np^2 = np(1-p)$	(M1) (A1)	3	Used; may be implied Or equivalent
(b)(i)	Mean = $np = 36$ SD = $\sqrt{np(1-p)} = 4.8$ Thus $36(1-p) = 4.8^2$ Thus $n = 100$ & $p = 0.36$	B1 M1 A1	3	Both CAO Attempt to solve for $p$ or $n$ Both CAO
(ii)	$P(30 < X < 40) =$  $P\left(Z < \frac{39.5-36}{4.8}\right) - P\left(Z < \frac{30.5-36}{4.8}\right) =$  $P(Z < 0.73) - P(Z < -1.15) =$  $P(Z < 0.73) - [1 - P(Z < 1.15)] =$  $0.76730 - [1 - (0.87286 \text{ to } 0.87493)] =$  $0.64 \text{ to } 0.643$	M1 B1 m1 A1	4	Standardising (39.5, 40 or 40.5) or (29.5, 30 or 30.5) with 36 and 4.8 and/or (36 - x) Use of 39.5 & 30.5 Area change AWFW (0.64112)
	<b>Total</b>		<b>10</b>	



**MS03 (cont)**

<b>Q</b>	<b>Solution</b>	<b>Marks</b>	<b>Total</b>	<b>Comments</b>		
<b>6(a)</b>	$E(X) = \underline{2.2}$	B1	3	CAO		
	$\text{Var}(X) = E(X^2) - 2.2^2 =$	M1		Used; or equivalent		
	$6.8 - 4.84 = 1.96$	A1		CAO		
<b>(b)(i)</b>	$E(S) = E(X) + 2.0 = 4.2$	B1F	5	F on (a)		
	$\text{Var}(S) = \text{Var}(X) + 1.5 + 2 \times (-0.43)$	M1		Used for $S$ or $D$		
	$= 2.6$	A1F		F on (a)		
<b>(ii)</b>	$E(D) = E(X) - 2.0 = 0.2$	B1F	5	F on (a)		
	$\text{Var}(D) = \text{Var}(X) + 1.5 - 2 \times (-0.43)$	A1F		F on (a)		
	$= 4.32$					
<b>(c)</b>	$L \sim N(2.31, 0.89^2) \quad M \sim N(2.04, 0.43^2)$	B1 B1	5		Both CAO; SD = 0.98843	
	$T = L + M \sim N(4.35, 0.977)$					
	$P(T > 5) = P\left(Z > \frac{5 - 4.35}{\sqrt{0.977}}\right)$			M1		Standardising 5 or 5.01 using C's mean & SD
	$= P(Z > 0.66) = 1 - P(Z < 0.66)$			m1		Area change
	0.25 to 0.26			A1		AWFW (0.25540)
	<b>Total</b>		<b>13</b>			

## MS03 (cont)

Q	Solution	Marks	Total	Comments
7	$X_D \sim \text{Po}(24)$			
(a)	$T = X_{\Sigma D} \sim \text{Po}(144)$	B1		CAO
Thus	$T \sim \text{approx } N(144, 144)$	M1		Normal with $\mu = \sigma^2$
	$P(T_{\text{Po}} \leq 150) \approx P(T_N < 150.5)$	B1		CAO
	$= P\left(Z < \frac{150.5 - 144}{12}\right)$	M1		Standardising (149.5, 150 or 150.5) with $\mu > 24$ and $\sqrt{\mu}$
	$= P(Z < 0.54) = 0.705 \text{ to } 0.71$	A1	5	AWFW (0.70598)
(b)(i)	$H_0: \lambda \text{ (or mean)} = 2 \text{ (or 10)}$	B1		Both; or equivalent
	$H_1: \lambda \text{ (or mean)} > 2 \text{ (or 10)}$			
	$P(Y \geq 17) = 1 - P(Y \leq 16)$	M1		Accept $1 - P(Y \leq 17)$
	$= 1 - 0.09730 = 0.027$	A1		AWRT
	$< 0.10 \text{ (10\%)}$	M1		Comparison of probability with 0.1
	$[z = 2.05 \text{ to } 2.38 > 1.2816]$			Comparison of $z$ with 1.2816 or 1.6449
	Thus evidence, at 10% level, of increase in mean daily number of requests	A1F	5	F on probability or on $z$
(ii)	CV of $Y$ is such that $P(Y \geq \text{CV}) \leq 0.10$ (10%)	M1		Can be implied by 13, 14 or 15 Accept $P(Y = \text{CV}) = 0.10$
Thus	$P(Y \leq \text{CV} - 1) \geq 0.90$	M1		Can be implied by 13, 14 or 15 Accept $P(Y = \text{CV}) = 0.90$
Thus	$\text{CV} = 15$	A1	3	CAO
(iii)	Power = $1 - P(\text{Type II error})$			Or equivalent
	= $1 - P(\text{accept } H_0 \mid H_0 \text{ false})$	B1		Stated or implied use
	= $P(\text{accept } H_1 \mid H_1 \text{ true})$			
	$\lambda = 5 \times 3 = 15$	B1		Stated or implied use of $\text{Po}(15)$
	Thus power = $P(Y \geq \text{CV})$	M1		Attempt at a probability based on C's CV from (ii) and $\text{Po}(15)$
	= $P(Y \geq 15) = 1 - P(Y \leq 14)$			
	= $1 - 0.4657 = 0.53 \text{ to } 0.54$	A1	4	AWFW (0.5343)
	<b>Total</b>		<b>17</b>	
	<b>TOTAL</b>		<b>75</b>	